J. Fluid Mech. (1999), *vol.* 381, *pp.* 225–238. Printed in the United Kingdom

Drag reduction of Newtonian fluid in a circular pipe with a highly water-repellent wall

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(Received 21 July 1997 and in revised form 17 August 1998)

Drag reduction phenomena, in which 14 % drag reduction of tap water flowing in a 16 mm-diameter pipe occurs in the laminar flow range, have been clarified. Experiments were carried out to measure the pressure drop and the velocity profile of tap water and an aqueous solution of glycerin flowing in pipes with highly water-repellent walls, by using a pressure transducer and a hot-film anemometer, respectively. The same drag reduction phenomena also occurred in degassed tap water when using a vacuum tank. The velocity profile measured in this experiment gives the slip velocity at the pipe wall, and it was shown that the shear stress is directly proportional to the slip velocity.

The friction factor formula for a pipe with fluid slip at the wall has been obtained analytically from the exact solution of the Navier–Stokes equation, and it agrees well qualitatively with the experimental data.

The main reasons for the fluid slip are that the molecular attraction between the liquid and the solid surface is reduced because the free surface energy of the solid is very low and the contact area of the liquid is decreased compared with a conventional smooth surface because the solid surface has many fine grooves. Liquid cannot flow into the fine grooves owing to surface tension. These concepts are supported by the experimental result that drag reduction does not occur in the case of surfactant solutions.

1. Introduction

For simple low-molecular fluids such as water or glycerin solutions, which are Newtonian fluids, fluid slip at the solid boundary is ordinarily negligible, and it is well known that the calculated result obtained under the no-slip boundary condition agrees well with the experimental results. However, if fluid slip occurs at the solid boundary, we can obtain a new drag reduction for Newtonian fluid flow. Although fluid slip is the most basic as well as a very interesting problem in fluid mechanics, very few studies on it have been performed. Watanabe *et al.* (1996) reported the laminar drag reduction of Newtonian fluids flowing in a channel for the first time, for square and rectangular ducts with highly water-repellent walls. The maximum drag reduction ratio of tap water was about 22 % for the square duct. Although the velocity profile was not measured, the friction factor was analysed by applying the fluid slip as the boundary condition, and the results were found to be in good agreement with those of experiments.

On the other hand, the same drag reduction phenomena were also observed for the frictional resistance of an enclosed rotating disk with a highly water-repellent wall (Watanabe & Ogata 1997). The maximum drag reduction ratio of tap water was about 25 % at the Reynolds number of about 2×10^5.

The purpose of this study is to clarify experimentally the characteristics of drag reduction in a circular pipe with water-repellent walls.

In the present study on pipe flow, experiments were carried out to measure the pressure drop and velocity profile of tap water and an aqueous solution of glycerin flowing in pipes with highly water-repellent walls, using a pressure transducer and a hot-film anemometer, respectively. Fluid slip of Newtonian fluids at the pipe wall was shown experimentally from a macroscopic point of view of fluid mechanics.

2. Analysis

The question of the conditions to be satisfied by a moving fluid in contact with a solid body was one of considerable difficulty for a long time, as pointed out by Goldstein (1965), and the assumption of no slip is now generally accepted for practical purposes. On the other hand, if we can make an artificial solid surface where there is very little interaction between the surface and the liquid in contact with it, slip would be appreciable for liquid flow.

Because fluid slip occurs at highly water-repellent walls when the contact angle is about 150°, we analyse the friction factor of slip flow in a circular pipe.

For a fully developed steady flow in a pipe, the Navier–Stokes equation can be written as

$$\frac{\mu}{r}\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u}{\mathrm{d}r}\right)\right] = \left(\frac{\mathrm{d}p}{\mathrm{d}z}\right). \tag{1}$$

By integrating this equation, and owing to the physical consideration that the velocity must be finite at $r = 0$, we obtain

$$u = \frac{r^2}{4\mu}\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right) + C_1.$$

The constant C_1 is evaluated under the boundary conditons at the pipe wall: $r = a$, $u = u_s$. Then, the slip velocity u_s is determined, using Navier's hypothesis, (Navier 1823; Goldstein 1965) from a macroscopic point of view:

$$\tau_w = \mu\left(-\frac{\mathrm{d}u}{\mathrm{d}r}\right)_{r=a} = \beta u_s, \tag{2}$$

where β is the sliding coefficient. For the case of $\beta \to \infty$, equation (2) agrees with the no-slip condition. Consequently, this gives

$$C_1 = \left(\frac{a}{2\beta} + \frac{a^2}{4\mu}\right)\left(-\frac{\mathrm{d}p}{\mathrm{d}z}\right), \tag{3}$$

and hence,

$$u = \left[\frac{a^2}{4\mu}\left(1 - \frac{r^2}{a^2}\right) + \frac{a}{2\beta}\right]\left(-\frac{\mathrm{d}p}{\mathrm{d}z}\right). \tag{4}$$

Concentration Cw(wt %)	Density ρ(kg m^{-3})	Viscosity μ(Pa s)	Temperature t(°C)
20	1.046×10^3	1.62×10^{-3}	25.0
30	1.070×10^3	1.93×10^{-3}	25.0

TABLE 1. Physical properties of the aqueous solutions of glycerin.

The volume flow rate is

$$Q = \int_0^a 2\pi r u \, \mathrm{d}r = \frac{\pi a^4 \Delta p}{8\mu l}\left(1 + \frac{4\mu}{a\beta}\right), \tag{5}$$

Where $(\Delta p/l)$ is the pressure gradient in a fully developed flow, and equals $(-\mathrm{d}p/\mathrm{d}z)$.

The friction factor for laminar flow λ is

$$\lambda = \frac{64}{Re} \frac{1}{\left[1 + \left(\dfrac{4\mu}{a\beta}\right)\right]}. \tag{6}$$

Thus, in laminar flow with fluid slip, the friction factor is a function of, not only the Reynolds number Re, but also the non-dimensional parameter $(\mu/a\beta)$.

If the flow does not exhibit fluid slip, equation (6) gives $\lambda = 64/Re$ on substituting $\beta \to \infty$ into the equation.

3. Experimental apparatus and method

Experiments were carried out to measure the pressure drop and the velocity profile of tap water and aqueous solution of 20–30 wt % glycerin in a circular pipe with a highly water-repellent wall.

The physical constants of test fluids are listed in Table 1. Test pipes are about 6 mm and 12 mm in diameter and 475 mm in length. Smooth pipes of the same size made of acrylic resin were used in order to compare the experimental results under no-slip conditons. The thickness of the highly water-repellent coating is less than 10 μm, and the diameter was determined as the mean value of the inlet and outlet diameters of the test pipe measured using a micrometer.

Figures 1 (*a*) and 1 (*b*) show the experimental circulation-type and the pressure-driven-type pipeline systems, respectively. In circulation-type system, test fluids were circulated by means of a centrifugal pump with variable rotational speed. Fully developed steady flow was obtained in the test section. The flow rate was measured by means of a digital counting scale. The details of the pressure hole and the traverser of a hot-film anemometer are shown in figures 2 (*a*) and 2 (*b*), respectively. The pressure difference at the test section was measured by means of a pressure transducer, and the measurement was repeated two or three times with increased flow rate.

The experimental pipeline system of the pressure-driven-type shown in figure 1 (*b*), was operated under the conditions of degassed water to study the effect of air in tap water on the drag reduction phenomena. Tap water in the pressure tank was degassed by a vacuum pump and the test liquid was held for about six hours in the vacuum. Subsequently, test liquids were forced through to the pipe by a compressor under

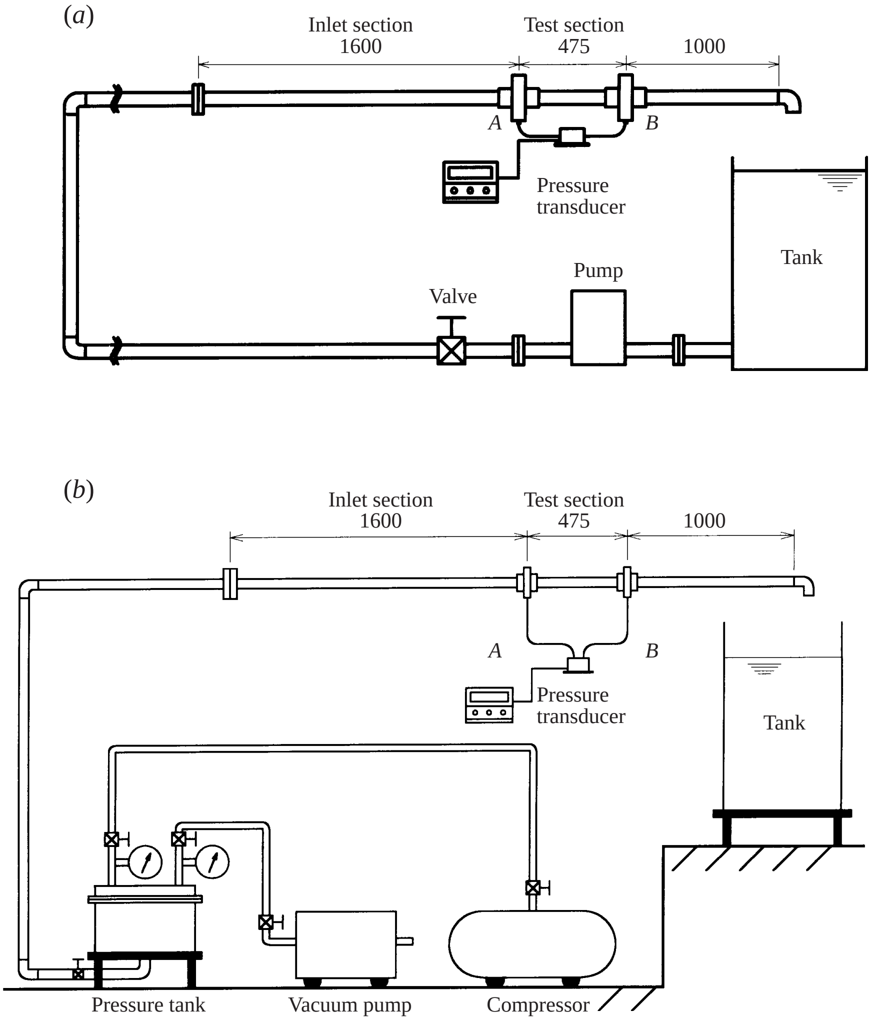


FIGURE 1. Experimental pipeline system. (*a*) Circulation system. (*b*) Pressure-driven system.

constant pressure. Two test liquids, which were set at −300 mmHg and −600 mmHg in the vacuum tank, were used. These liquids were discarded after the pressure drop in the test section was measured. The test pipe was 6 mm in diameter.

Figures 3 (*a*) and 3 (*b*), respectively, show the shapes of a drop of water on the highly water-repellent wall and acrylic resin wall used in this study. The highly water-repellent coating was the same as that used previously for the ducts (Watanabe *et al.* 1996). The contact angle with the wall is about 150°. In general, the largest contact angles recorded for a smooth surface are 112°–115° (Moilliet 1963). Thus, we need a useful method of producing a surface with a contact angle of 120° or larger, since it is necessary not only to reduce the free surface energy but also to change the surface morphology.

Figures 4 (*a*) and 4 (*b*) show micrographs of the tested highly water-repellent wall observed using a microscope and an SEM, respectively. There are many fine grooves on the surface that raise the water repellency. The basic material of the

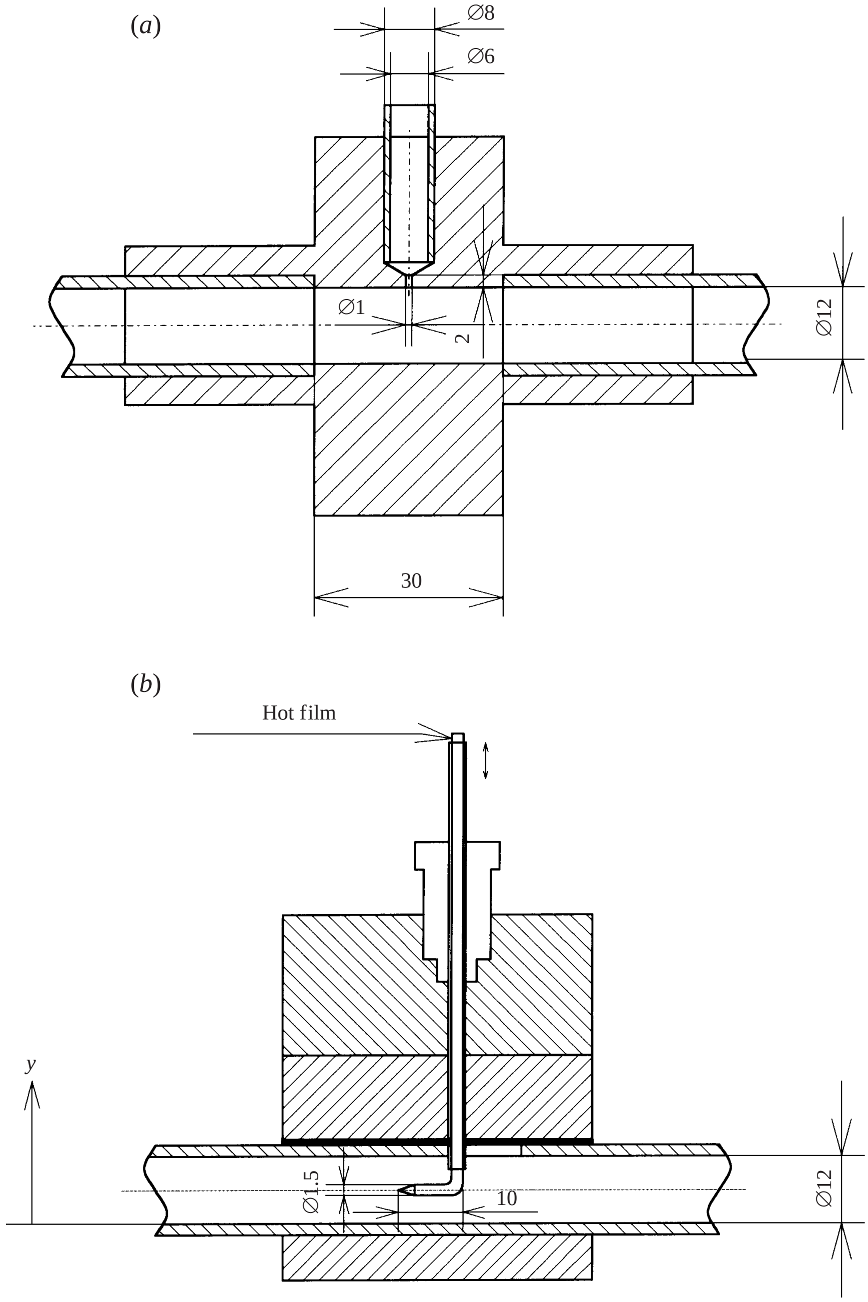


FIGURE 2. Details of test section. (*a*) Pressure hole. (*b*) Hot-film anemometer.

highly water-repellent coating is fluorine alkane modified acrylic resin with added hydrophobic silica, which was left overnight in air after it was coated to the pipe wall.

The velocity profile was measured in the region between the wall and the centre of the pipe.

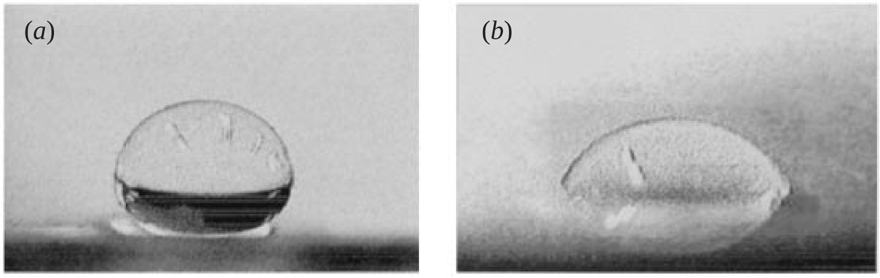


FIGURE 3. Shapes of a drop of water on a wall. (*a*) Highly water-repellent wall. (*b*) Smooth aluminium wall.

4. Results and discussion

4.1. *Pressure drop*

Figures 5(*a*)–5(*c*) show the experimental results for the friction factor obtained using the circulation-pipeline-system. In these figures, the solid lines indicate the analytical results for laminar flow, $\lambda = 64/Re$, and the Blasius correlation for turbulent flow, $\lambda = 0.3164\, Re^{-0.25}$. Experimental data for tap water, and 20 and 30 wt % glycerin solutions in acrylic resin pipes fit these lines in laminar and turbulent flow ranges. The reported value for tap water in the laminar flow range in an acrylic resin pipe is the best estimate of the result, and with 95 % confidence, the true value is believed to lie within 3.15 % of it.

On the other hand, the experimental data for pipes with highly water-repellent walls indicate drag reduction in the laminar flow range, and the friction factor, λ, is proportional to Re^{-1} . The experimental data for the glycerin solutions show a similar tendency.

Figure 6 shows the experimental results for the friction factor obtained using the pressure-driven-pipeline system. The dotted lines in the figure indicate the values obtained for the 6 mm pipe in the circulation-pipeline-system. The values measured under three conditions almost agree. The drag reduction occurs in the laminar flow region and the experimental data fit the dotted line. Thus, it can be concluded that degassing the water has no effect on drag reduction.

Figure 7 shows the relationship between the drag reduction ratio and the concentration of glycerin solution. The drag reduction ratio is defined as

$$\mathrm{DR} = \left| \frac{\lambda r - \lambda a}{\lambda a} \right| \times 100\ (\%), \tag{7}$$

where λr and λa are the friction factors of pipes with a highly water-repellent wall and an acrylic resin wall, respectively. Drag reduction is affected significantly by the diameter of the pipe and fluid viscosity. This relation may be inferred from equation (6). However, in order to calculate the friction factor using equation (6), it is necessary to know the physical constant β. The method of determining β makes use of equation (2) to correct the experimental data of the velocity profile and the pressure drop.

The transition Reynolds number of the pipe with a highly water-repellent wall increases slightly. At the transition, the friction factor curve increases at a lower rate than that of the smooth pipe, and finally merges with the curve for the smooth pipe at large Reynolds numbers, i.e. in the turbulent flow range. The trend is similar to that in rectangular ducts. It is not clear at present why drag reduction does not occur

(*a*)

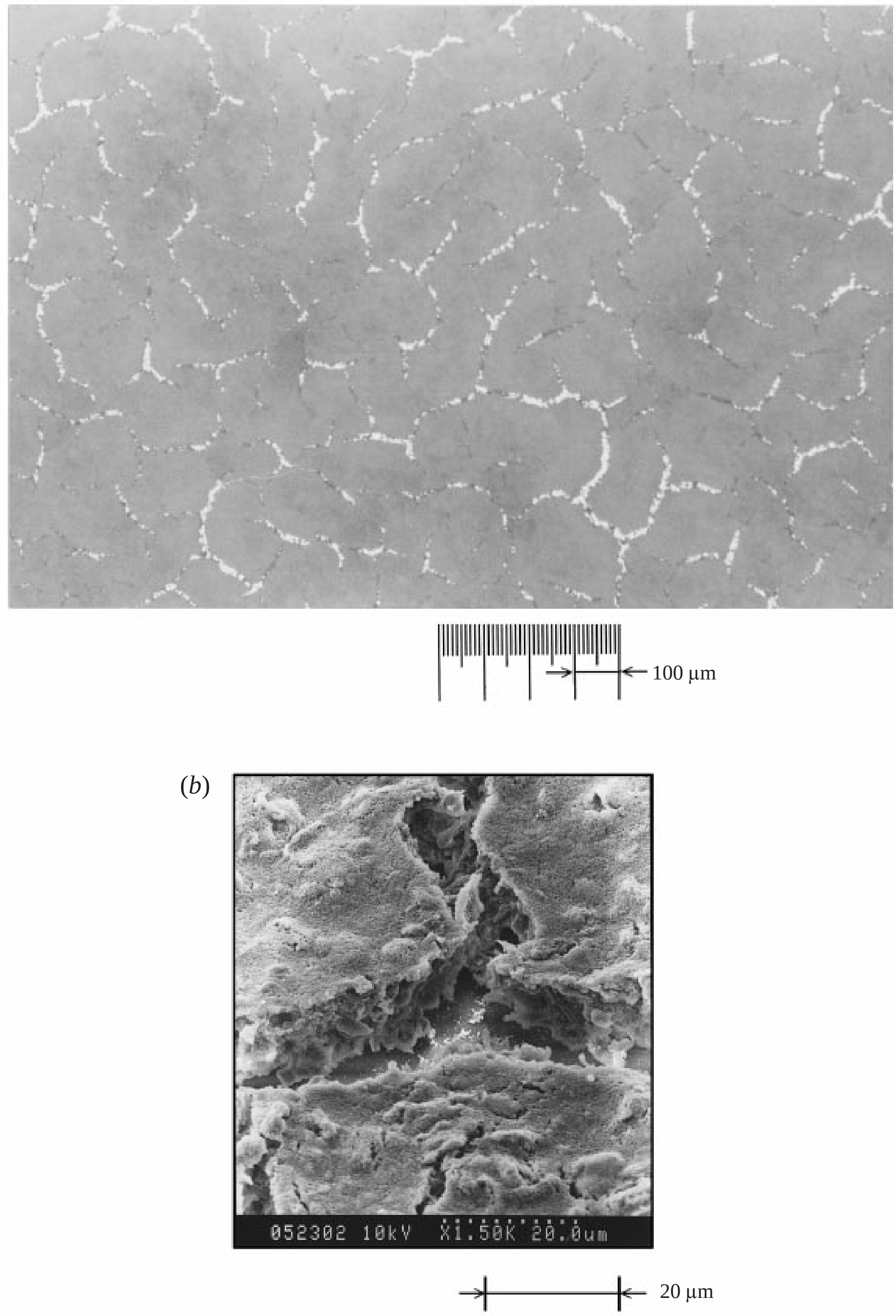
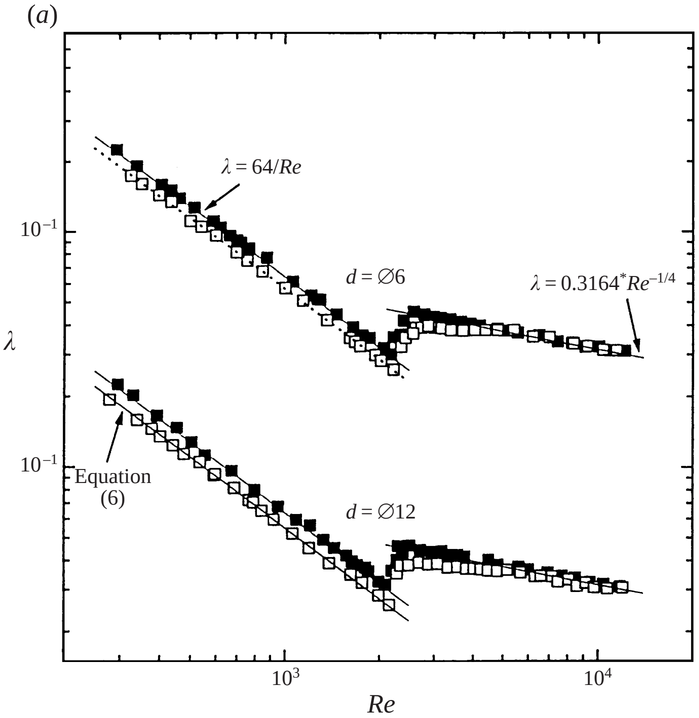


FIGURE 4. Micrographs of the highly water-repellent wall. (*a*) Observed by a microscope. (*b*) Observed by SEM.



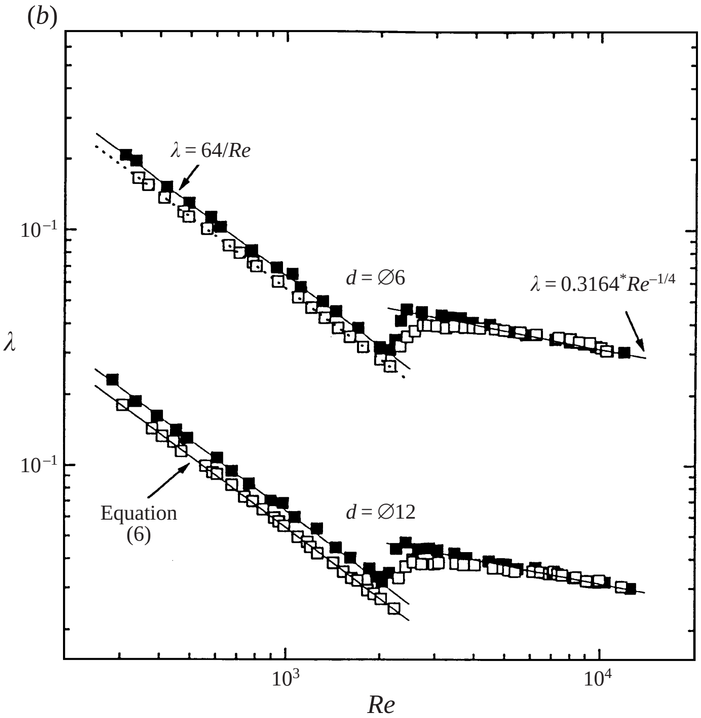


FIGURE 5(a, b). For caption see facing page.

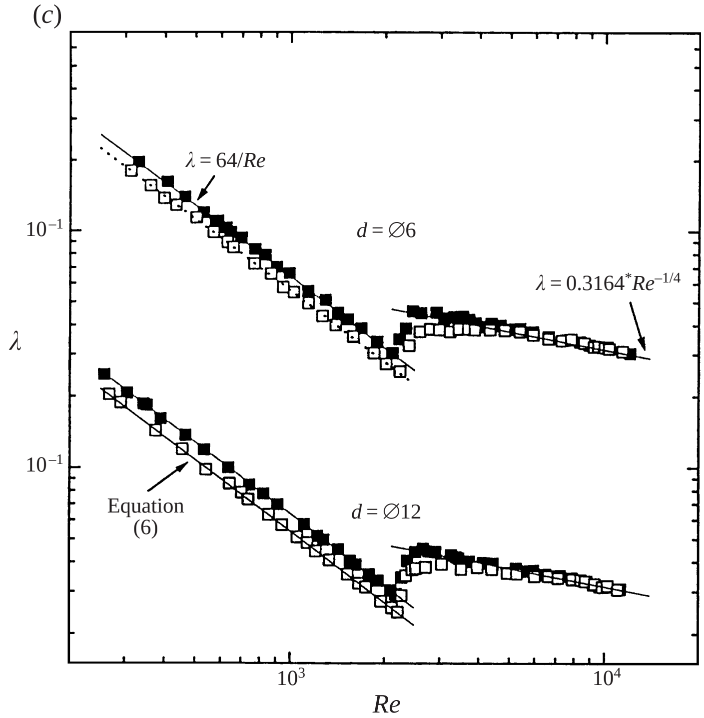


FIGURE 5. Friction factor. ■, for acrylic resin wall; □, for highly water-repellent wall. (*a*) Tap water. (*b*) Glycerin 20 wt % solution. (*c*) Glycerin 30 wt % solution.

in the turbulent flow region. It will be necessary to obtain experimental data near the wall in more detail to elucidate the mechanism of fluid slip.

4.2. *Velocity profile*

In spite of the simplicity of the flow field, no data have been published for the slip of Newtonian fluids in a circular pipe. Figure 8 shows the velocity profile of tap water for fully developed laminar flow through a pipe with a highly water-repellent wall. The velocity profile through a smooth pipe is also shown for comparison under the same pressure gradient condition, $-(\mathrm{d}p/\mathrm{d}z) = 22.6\,\mathrm{Pa}\,/m^{-1}$. Experimental data for the smooth pipe fit the exact solution of the Navier–Stokes equation, that is the flow was Hagen–Poiseuille flow.

However, the data for the pipe with a highly water-repellent wall does not fit and the velocity increases. This indicates that drag reduction occurs in this region.

The drag reduction ratios obtained by integrating the velocity profile in figure 8 and by measuring the pressure drop, respectively, are 13.8 % and 14.0 %. These two values almost agree.

By extrapolating the experimental data of the velocity profile, the slip velocity at the pipe wall is confirmed. The effect of Reynolds number in the velocity profile is also illustrated in figures 9 (*a*)–9 (*c*). With the increase of Reynolds number, the slip velocity increases in the two cases of tap water and glycerin solutions. At a given Reynolds number, the wall shear stress is calculated, and we can estimate the physical

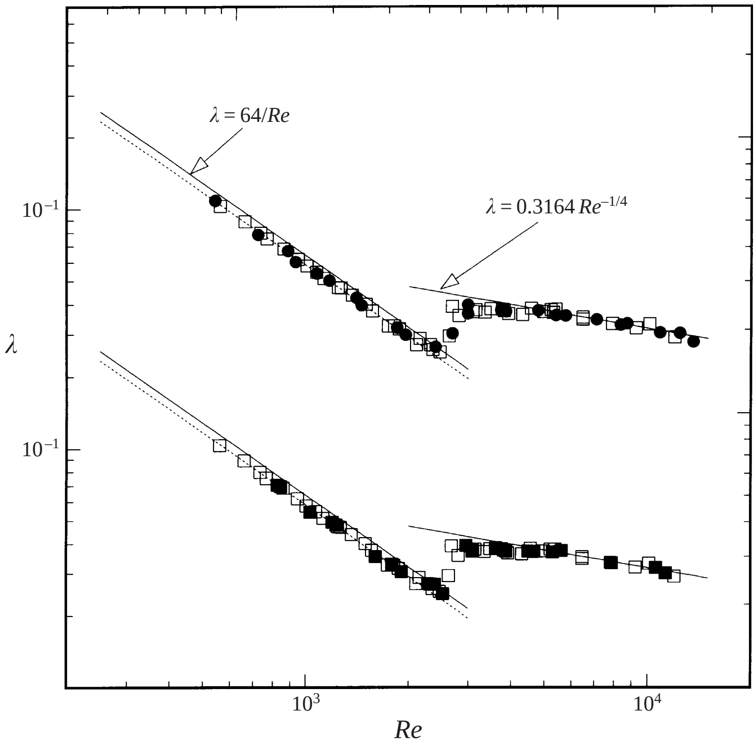


FIGURE 6. Friction factor in pipe of $d = 6$ mm with highly water-repellent wall. □, atmospheric pressure; ●, −300 mmHg; ■, −600 mmHg.

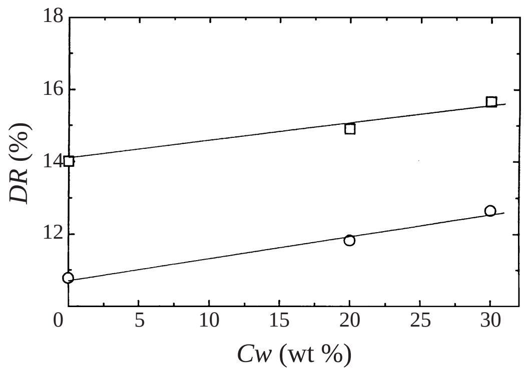


FIGURE 7. Drag reduction ratio obtained for pipes of □, $d = 12$ mm and ○, $d = 6$ mm.

constant β using equation (2) from the relationship between the wall shear stress τ_w and the slip velocity u_s, as mentioned above.

Figure 10 shows the relationship between τ_w calculated using the experimental data of the pressure and u_s in figures 9 (*a*)–9 (*c*). The relationship indicates that the slip velocity is directly proportional to the wall shear stress; the sliding coefficient β is a constant value. These results are important, since they experimentally confirm, for the first time, equation (2) proposed by Navier.

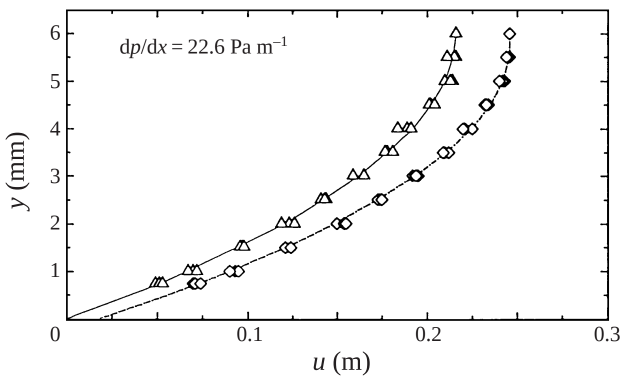


FIGURE 8. Comparison between velocity profiles of tap water in pipes with △, acrylic resin; and ◇, highly water-repellent walls.

Figure 11 shows the relationship between the sliding coefficient and viscosity. The data extrapolated from the friction factor of the square duct (Watanabe *et al.* 1996) are also shown for comparison. β increases slightly with the increase of viscosity.

In figures 5 (a)–5 (c), the results for equation (6), calculated by substituting the value of β, are represented by the solid lines. It is convenient to consider the drag reduction in the relationship between $(\lambda - Re)$ and the non-dimensional parameter $S = (\mu/a\beta)$, because the friction factor depends on Re and S.

The results are shown in figure 12 which indicates excellent agreement between calculated and experimental values of the friction factor over the laminar flow range. However, no value of β obtained in this study is applicable in equation (6) for the case of a pipe diameter of 6 mm, because it is an inherent value for this case. It can be considered that β depends directly on not only the properties of the solid boundary and the physical constant of the fluid, but also the characteristic length of the flow field, for example, the pipe diameter.

There is no completely satisfactory and general method available for the prediction of β. Because fluid slip of Newtonian fluid does not occur in a smooth Teflon pipe, e.g. when the contact angle is less than about 110°, whereas it occurs at the highly water-repellent wall for which the contact angle is about 120°–150°, the contact angle will be a significant physical constant when considering the relationship of β to fluid slip.

On the other hand, the mechanism of fluid slip at the highly water-repellent wall is more complex, and not yet well understood. As is evident in figures 4 (a) and 4 (b), the surface has a porous structure. Thus, the effect of an interaction at the air–liquid interface on the flow behaviour near the wall cannot be neglected when studying fluid flow from the microscopic viewpoint.

However, it has been shown that drag reduction does not occur for a surfactant solution in a circular pipe with highly water-repellent walls, although it does for a dilute polymer solution (Watanabe & Udagawa 1998).

Thus, it can be considered that tap water or glycerin solution does not contact the surface of the fine groove because of surface tension, and the air between the liquid surface and the groove of the wall plays an important role in fluid slip. It will be necessary to study the flow behaviour near the solid wall in detail.

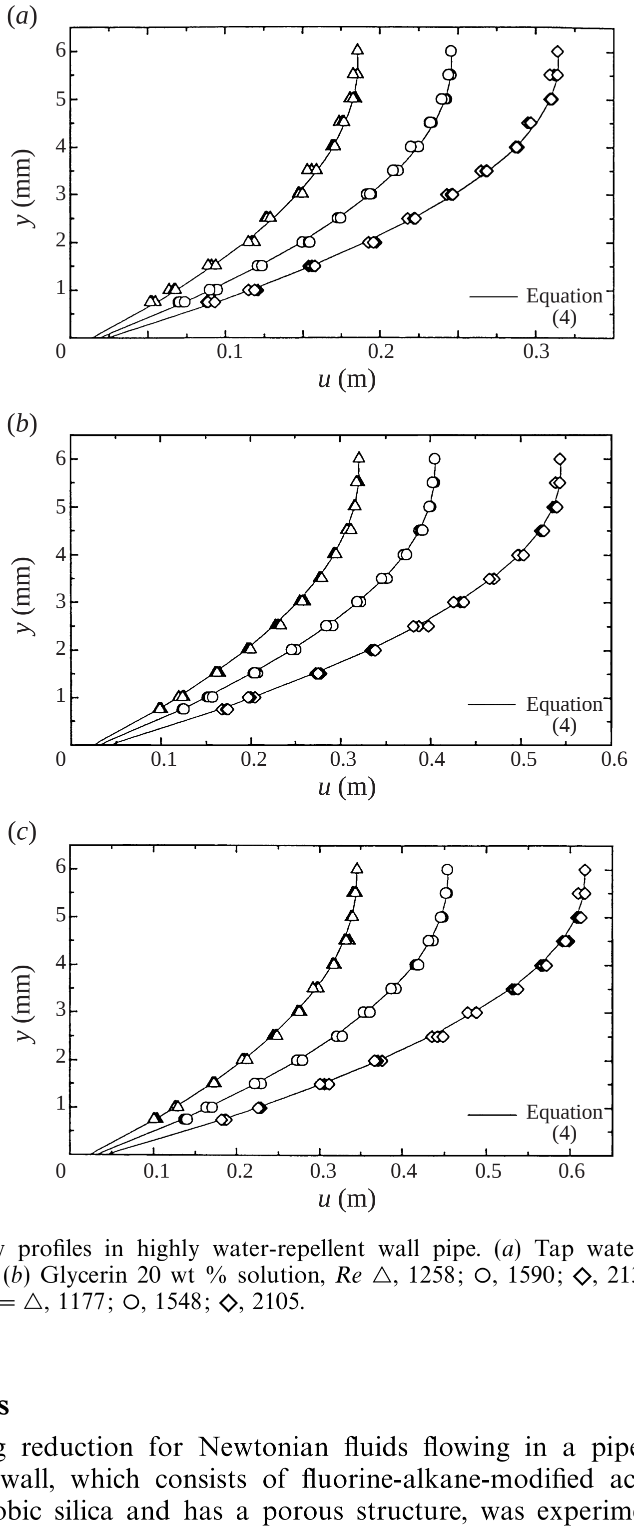


FIGURE 9. Velocity profiles in highly water-repellent wall pipe. (*a*) Tap water, *Re* = △, 1250; ○, 1650; ◇, 2100. (*b*) Glycerin 20 wt % solution, *Re* △, 1258; ○, 1590; ◇, 2137. (*c*) Glycerin 30 wt % solution, *Re* = △, 1177; ○, 1548; ◇, 2105.

5. Conclusions

Laminar drag reduction for Newtonian fluids flowing in a pipe with a highly water-repellent wall, which consists of fluorine-alkane-modified acrylic resin with added hydrophobic silica and has a porous structure, was experimentally clarified, and the fluid slip velocity was measured by means of a hot-film anemometer. The results obtained are summarized as follows.

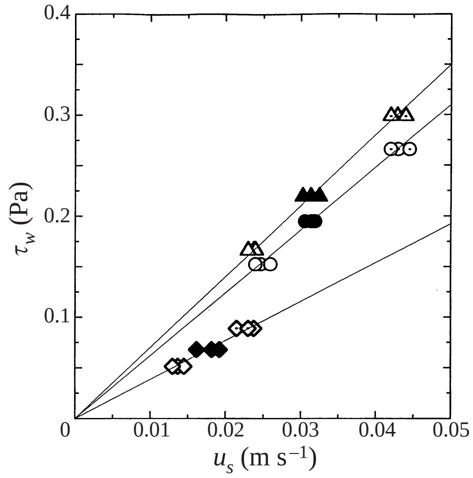


FIGURE 10. Relationship between wall shear stress and slip velocity for tap water (◇, Re = 1250; ◆, 1650; ◈, 2100), glycerin 20 wt % (○, Re = 1250; ●, 1580; ⊙, 2130), and glycerin 30 wt % (△, Re = 1170; ▲, 1540; ◬, 2100).

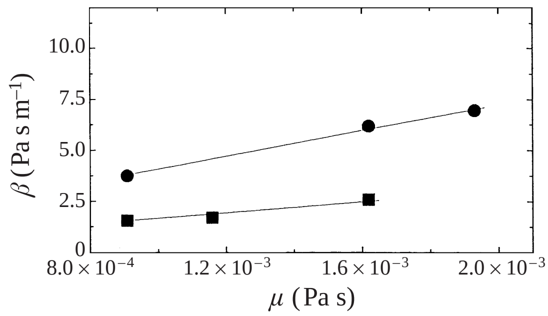


FIGURE 11. Sliding coefficient for Newtonian fluids. ●, present study for pipe of $d = 12$ mm; ■, Watanabe *et al.* for a square duct of $a = 15$ mm.

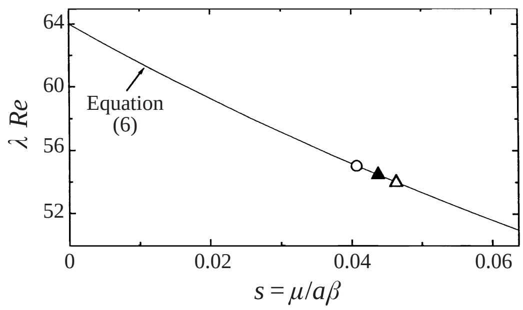


FIGURE 12. Friction factor vs. parameter ($\mu/a\beta$) for pipe of $d = 12$ mm. ○, $Cw = 0$ wt %; ▲, 20 wt %; △, 30 wt %.

(i) The laminar drag reduction ratio for tap water flowing in a 12-diameter mm pipe, is about 14 % and it increases with increasing viscosity.

(ii) Values obtained using equations (6) and (4) agree well with the experimental results for the friction factor and the velocity profile, respectively.

(iii) The wall shear stress is proportional to the slip velocity, and β in equation (2) depends directly on not only the properties of the solid boudary and the fluid but also the characteristic length of the flow field.

We thank E. Nakatani, Y. Doi and N. Harutani of Kansai Paint Co. Ltd, for their support in setting up the experimental apparatus. This work was supported financially by the Scientific Research Fund of the Ministry of Education of Japan (B-09450082).

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